REGNECENTRALEN

DANSK INSTITUT FOR MATAMATIKMASKINER

DASK - BIBLIOTEKSSPECIFIKATION

SEKVENS-BETEGNELSE
K.A. nr. 1
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Kodet af K.A.

Indkørt af K.A. og P.N.

Udgivet d. 13.8.1960

Besselfunktionerne

I0, K0, I1, K1

Indhops-adresser	Udhops-adresse	Indgang	Udgang
0A8	186A8	C(FAR) = x	I0(x) → 0AA K0(x) → 2AA I1(x) → 4AA K1(x) → 6AA flydende pakket

Kodelængde 0 - 313	Undersekvenser FR 1 i 0A9 AF 1 i 0AC LF 1 i 0AB XF 1 i 0AD
Begyndelsesadresse lige	Arbejdsceller FMD, FAR, FMR i sekvensen 0AA, 1AA, 4AA, 5AA
Grundparametre E9, EB, EC, ED undersekv. EA arbc. og udg.	Perm. konstanter 2039; 2041-2043
Programparametre ingen	

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Grundlag

Sekvensen udregner de to Besselfunktioner In(x) og Kn(x) som funktioner af x og for faste n: n = 0 og n = 1 efter følgende formler:

(Se "Mathematical Tables and other Aids to Computation" Vol 10 162 - 164)

$$I0(x) = P1(x) \qquad \text{for } x \leq 3.75$$
$$I1(x) = P2(x) \times x$$

$$I0(x) = P3(x) \times \exp(x)/\sqrt{x} \qquad \text{for } x \geq 3.75$$
$$I1(x) = P4(x) \times \exp(x)/\sqrt{x}$$

$$K0(x) = P5(x) - \ln(x/2) \times I0(x) \qquad \text{for } x \leq 2$$
$$K1(x) = P6(x)/x + \ln(x/2) \times I1(x)$$

$$K0(x) = P7(x)/\exp(x) \times \sqrt{x} \qquad \text{for } x \geq 2$$
$$K1(x) = P8(x)/\exp(x) \times \sqrt{x}$$

hvor

$$P1(x) = 1 + 3.5156229 \times (x/3.75)\uparrow 2 + 3.0899424 \times (x/3.75)\uparrow 4 + 1.2067492 \times (x/3.75)\uparrow 6 + .2659732 \times (x/3.75)\uparrow 8 + .0360768 \times (x/3.75)\uparrow 10 + .0045813 \times (x/3.75)\uparrow 12$$

$$P2(x) = .5 + .87890594 \times (x/3.75)\uparrow 2 + .51498869 \times (x/3.75)\uparrow 4 + .15084934 \times (x/3.75)\uparrow 6 + .02658733 \times (x/3.75)\uparrow 8 + .00301532 \times (x/3.75)\uparrow 10 + .00032411 \times (x/3.75)\uparrow 12$$

$$P3(x) = .398942280 + .013285917 \times (3.75/x) + .002253187 \times (3.75/x)\uparrow 2 - .001575649 \times (3.75/x)\uparrow 3 + .009162808 \times (3.75/x)\uparrow 4 - .020577063 \times (3.75/x)\uparrow 5 + .026355372 \times (3.75/x)\uparrow 6 - .016476329 \times (3.75/x)\uparrow 7 + .003923767 \times (3.75/x)\uparrow 8$$

$$P4(x) = .398942280 - .039880242 \times (3.75/x) - .003620183 \times (3.75/x)\uparrow 2 + .001638014 \times (3.75/x)\uparrow 3 - .010315550 \times (3.75/x)\uparrow 4 + .022829673 \times (3.75/x)\uparrow 5 - .028953121 \times (3.75/x)\uparrow 6 + .017876535 \times (3.75/x)\uparrow 7 - .004200587 \times (3.75/x)\uparrow 8$$

$$P5(x) = -.57721566 + .42278420\times(x/2)\uparrow 2 + .23069756\times(x/2)\uparrow 4 + .03488590\times(x/2)\uparrow 6 + .00262698\times(x/2)\uparrow 8 + .00010750\times(x/2)\uparrow 10 + .00000740\times(x/2)\uparrow 12$$

$$P6(x) = 1 + .15443144\times(x/2)\uparrow 2 - .67278579\times(x/2)\uparrow 4 - .18156897\times(x/2)\uparrow 6 - .01919402\times(x/2)\uparrow 8 - .00110404\times(x/2)\uparrow 10 - .00004686\times(x/2)\uparrow 12$$

$$P7(x) = 1.25331414 - .07832358\times(2/x) + .02189568\times(2/x)\uparrow 2 - .01062446\times(2/x)\uparrow 3 + .00587872\times(2/x)\uparrow 4 - .00251540\times(2/x)\uparrow 5 + .00053208\times(2/x)\uparrow 6$$

$$P8(x) = 1.25331414 + .23498619\times(2/x) - .03655620\times(2/x)\uparrow 2 + .01504268\times(2/x)\uparrow 3 - .00780353\times(2/x)\uparrow 4 + .00325614\times(2/x)\uparrow 5 - .00068245\uparrow(2/x)\uparrow 6$$

Metoden, som den er angivet i "Mathematical Tables and other Aids to Computation" tillader, at I0(x), K0(x), I1(x) og K1(x) beregnes med en nøjagtighed paa 7 - 8 betydende cifre.

I sekvensen er polynomiets koefficienter indlæst med en skalafaktor 1/4, saaledes at der aldrig opstaar spild ved beregning af et polynomium. De beregnede polynomier omskrives til tal paa flydende, pakket form. Derefter udregnes funktionerne udelukkende ved hjælp af flydende tal. Sekvensen giver resultater, der stemmer overens med tabelværdier i 7 eller 8 cifre.

Er $x \leq 0$, stopper sekvensen absolut i 46A9 i FR 1.

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Algol-program.

(Grundlag: Algol 60 rapport)

```
begin
real    I0, K0, I1, K1;
comment x er erklæret i hovedprogrammet,
der benytter Besselkoden;
procedure pol(a0, a1, a2, a3, a4, a5, a6, a7, a8, z, p);
real a0, a1, a2, a3, a4, a5, a6, a7, a8, z, p;
p:= (((((((a8×z + a7)×z + a6)×z + a5)×z + a4)×z + a3)×z + a2)×z + a1)×z
                                                          + a0)
if x < 0 then go to stop;
if x = 0 then go to stop;
comment i det nuværende program stoppes absolut paa x<0 og x=0;
if x > 2 then begin pol(1.25331414, -.07832358, .02189568, -.01062446
                .00587872, -.00251540, .00053208, 0, 0, 2/x, P7);
                K0:= P7/sqrt(x)×exp(x);
                pol(1.25331414, .23498619, -.03655620, .01504268
                -.00780353, .00325614, -.00068245, 0, 0, 2/x, P8);
                K1:= P8/sqrt(x)×exp(x);
                if x > 3.75 then begin pol(.398942280, .013285917,
                                .002253187, -.001575649, .009162808,
                                -.020577063, .026355372, -.016476329,
                                .003923767, 3.75/x, P3);
                                I0:= P3×exp(x)/sqrt(x);
                                pol(.398942280, -.039880242, -.003620183,
                                .001638014, -.010315550, .022829673,
                                -.028953121, .017876535, -.004200587
                                3.75/x, P4);
                                I1:= P4×exp(x)/sqrt(x);
                                go to stop;
                                end comment x>3.75;
              end comment x>2;
pol(1, 3.5156229, 3.0899424, 1.2067492, .2659732, .0360768, .0045813,
0, 0, (x/3.75)↑2, P1);
I0:= P1;
pol(.5, .87890594, .51498869, .15084934, .02658733, .00301532, .00032411,
0, 0, (x/3.75)↑2, P2);
I1:= P2×x;
if x > 2 then go to stop;
pol(1, .15443144, -.67278579, -.18156897, -.01919402, -.00110404,
-.00004686, 0, 0, (x/2)↑2, P6);
K1:= P6/x + I1×ln(x/2);
pol(-.57721566, .42278420, .23069756, .03488590, .00262698, .00010750,
.00000740, 0, 0, (x/2)↑2, P5);
K0:= P5 - I0×ln(x/2);
stop: ;
end
```

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```
Indhop ->    0    164 A8 34   P:= IRB
             1    165 A8 54   Q:= IRC
             2    166 A8 74   R= IRD
             3   2000 A  40 ┐
             4     46 A9 51 ├ stop hvis x=0
             5   2041 A  01 ├ stop hvis x<0
             6     46 A9 51 ┘
             7     41 A9 16   FMD:= x=x' x2↑(x''-1024)
             8   2039 A  61 ┐
             9     32 A9 16 ┘ FAR:= x/2=x' x2↑(x''-1024-1)
            10   2003 A  61 ┐
            11   2043 A  20 ├ S:= (-x''+1024+1)x2↑(-11)
            12    124 A8 29 ┘
  <-        13     93 A811   hop til A hvis S>0
hvis x>2    14   2003 A  60   AR:= (x''-1)x2↑(-11)
            15     49 A9 21   AR:= (x'' -1026)x2↑(-11)
            16     21 A8 29   x'' -1026 lagres
            17   2043 A  60   AR:= 2↑(-1)
            18   2041 A  01   AR:= 2↑(-1)-2↑(-39)
            19   2000 A  0B   MR:= (2↑(-1)-2↑(-39))/x'
  <-        20     21 A8 50   tøm AR og hop til næste ordre
20 ->       21    (0) A  4D   MR:= (2↑(-1)-2↑(-39))/x' x2↑(-x''+1026)
            22      0 A  07   AR:=MR=2/x
            23    188 A8 08   z:= 2/x
            24   2039 A  60 ┐
            25   2003 A  26 ┘ FAR:= x
            26     26 AC 16   FAR:= sqrt(x)
            27   2016 A  16 ┐
            28      0 AA 08 ┘ I0:= sqrt(x)
            29   2000 A  40 ┐
            30   2004 A  08 │
            31   2003 A  60 ├ FMR:= sqrt(x)
            32   2007 A  28 ┘
            33   1996 A  40 ┐
            34   2000 A  08 │
            35   1999 A  60 ├ FAR:= x
            36   2003 A  28 ┘
            37     62 AD 16 ┐
            38      2 A  00 ┘ FAR:= exp(x)
            39   2016 A  16 ┐
```

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```
                40      4 AA 08   I1:= exp(x)
                41     57 A9 16   FAR:= sqrt(x)×exp(x)
                42   1996 A  40
                43   2004 A  08
                44   1999 A  60   FMR:= x
                45   2007 A  28
                46     41 A9 16   FMD:= sqrt(x)×exp(x)
                47    296 A8 55   IRC:= 294A8+2
          <-    48    168 A8 16   hop til p, FAR:= P7
186 ->          49     50 A9 16   FAR:= P7/(sqrt(x)×exp(x))
                50   2016 A  16
                51      2 AA 08   K0:= P7/(sqrt(x)×exp(x))
                52    310 A8 55   IRC:= 308A8+2
          <-    53    168 A8 16   hop til p, FAR:= P8
186 ->          54     50 A9 16   FAR:= P8/(sqrt(x)×exp(x))
                55   2016 A  16
                56      6 AA 08   K1:= P8/(sqrt(x)×exp(x))
                57   2004 A  40
                58   1996 A  08
                59   2007 A  60   FMD:= x
                60   1999 A  28
                61    310 A8 40
                62   2026 A  16   FAR:= 3.75
                63     50 A9 16   FAR:= 3.75/x=q'↑(q''1024)
                64   2003 A  61
                65   2043 A  20   AR:= (-q''+1024)×2↑(-11)
          <-    66     93 A8 51   hop til A hvis AR<0
hvis x>3.75     67     69 A8 29
                68   2000 A  40
                69    (0) A  0D   z:= 3.75/x
                70    188 A8 08
                71      4 AA 40
                72   2026 A  16   FAR:= exp(x)
                73      0 AA 40
                74   2021 A  16   FMD:= sqrt(x)
                75     50 A9 16   FAR:= exp(x)/sqrt(x)
                76   2000 A  40
                77   2004 A  08
                78   2003 A  60   FMR:= exp(x)/sqrt(x)
                79   2007 A  28
```

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```
                80     18 A  35   IRB:= 18
                81    236 A8 55   IRC:= 234A8+2
     <-         82    169 A8 16   hop til p, FAR:= P3
186 ->          83     57 A9 16   FAR:= P3x(exp(x)/sqrt(x))
                84   2016 A  16 ]
                85      0 AA 08 ] IO:= P3x(exp(x)/sqrt(x))
                86     18 A  35   IRB:= 18
                87    254 A8 55   IRC:= 252A8+2
     <-         88    169 A8 16   hop til p1, FAR:= P4
186 ->          89     57 A9 16   FAR:= P4x(exp(x)/sqrt(x))
                90   2016 A  16 ]
                91      4 AA 08 ] I1:= P4x(exp(x)/sqrt(x))
     <-         92    164 A8 10   hop til E, slut x>3.75 og x>2
A 13, 66 ->     93   1996 A  40 ]
                94   2000 A  08 |
                95   2004 A  08 | FAR:= x
                96   1999 A  60 | FMR:= x
                97   2003 A  28 |
                98   2007 A  28 ]
                99    312 A8 40 ]
               100   2021 A  16 ] FMD:= 3.75000003
               101     50 A9 16 ]
               102   2003 A  61 |
               103   2043 A  20 |
               104    106 A8 29 |
               105   2000 A  40 | z:= 3.75000003↑2
               106    (0) A  0D |
               107    188 A8 08 |
               108   2042 A  24 |
               109    188 A8 0A |
               110    188 A8 08 ]
               111    204 A8 55   IRC:= 202A8+2
     <-        112    168 A8 16   hop til p, FAR:= P1
186 ->         113   2016 A  16 ]
               114      0 AA 08 ] IO:= P1
               115    218 A8 55   IRC:= 216A8+2
     <-        116    168 A8 16   hop til p, FAR:= P2
186 ->         117     57 A9 16   FAR:= xxP2
               118   2016 A  16 ]
               119      4 AA 08 ] I1:= xxP2
```

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```
                120    124 A8 60   AR:= S
(-              121    164 A8 51   hop til E hvis x>2
                122   2004 A  40 ┐
                123   1996 A  08 │
                124    (0) A  0D │
                125    188 A8 08 │ z:= (x/2)↑2
                126   2042 A  24 │ FMD:= x
                127    188 A8 0A │
                128    188 A8 08 │
                129   2007 A  60 │
                130   1999 A  28 ┘
                131    282 A8 55   IRC:= 280A8+2
(-              132    168 A8 16   hop til p, FAR:= P6
186 ->          133     50 A9 16   FAR:=P6/x
                134     41 A9 16   FMD:= P6/x
                135   2004 A  40 ┐
                136   2000 A  08 │
                137   2007 A  60 │ FAR:= x/2
                138   2039 A  21 │
                139   2003 A  28 ┘
                140     56 AB 16 ┐
                141      2 A  00 ┘ FAR:= ln(x/2)
                142   2000 A  40 ┐
                143   2004 A  08 │
                144   2003 A  60 │ FMR:= ln(x/2)
                145   2007 A  28 ┘
                146      4 AA 40 ┐
                147   2026 A  16 ┘ FAR:= I1
                148     57 A9 16   FAR:= I1×ln(x/2)
                149      2 A9 16   FAR:= I1×ln(x/2)+P6/x
                150   2016 A  16 ┐
                151      6 AA 08 ┘ K1:= I1×ln(x/2)+P6/x
                152      0 AA 40 ┐
                153   2026 A  16 │
                154   2000 A  41 │ FAR:= -I0
                155   2036 A  16 │
                156   2000 A  08 ┘
                157     57 A9 16   FAR:= -I0×ln(x/2)
                158     41 A9 16   FMD:= -I0×ln(x/2)
                159    268 A8 55   IRC:= 266A8+2
```

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```
   <-       160    168 A8 16   hop til p, FAR:= P5
186 ->      161      2 A9 16   FAR:= P5-I0xln(x/2)
            162   2016 A  16 ]
            163      2 AA 08 ]  K0:= P5-I0xln(x/2)
E 92,121->  164    (0) A  35   IRB:= P
            165    (0) A  55   IRC:= Q
            166    (0) A  75   IRD:= R
            167      1 D  10   udhop
p 48,53 ]
112,116 ] -> 168     14 A  35   IRB:= 14
132,160 ]
p1 82,88 <-> 169    170 A8 50   AR:= 0
R 176,169-> 170    2046 B  35   IRB:= IRB-2
            171    2046 C  55   IRC:= IRC-2
            172       0 C  04   AR og MR:= AR+i'te koeff. i pol.
   <-       173    175 A8 33 ]
   <-       174    177 A8 10 ]  hop til L hvis IRB=0
173 ->      175    188 A8 0A   AR:=MRxz
   <-       176    170 A8 10   hop til R
L 174 ->    177   2003 A  0E ]
            178   2000 A  08
            179   2000 A  43
            180    186 A8 11
            181   2039 A  60   FAR:= pol.
            182      2 A  0C
            183   2043 A  20
            184   2003 A  21
            185   2003 A  28 ]
   <-       186      1 D  10

            188 ]
            189 ]      z

                  1844 E  3
                  1020 A  00   sæt skalafaktorer i indlæseprogram
                   190 EE 3

            190
                  C1CA
            191

            192
                  C3C5156229A
            193

            194
                  C3C0899424A
            195

            196
                  C1C2067492A
            197
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198	
	CC2659732A
199	
200	
	CC0360768A
201	
202	
	CC0045813A
203	
204	
	CC5A
205	
206	
	CC87890594A
207	
208	
	CC51498869A
209	
210	
	CC15084934A
211	
212	
	CC02658733A
213	
214	
	CC00301532A
215	
216	
	CC00032411
217	
218	
	CC398942280A
219	
220	
	CC013285917A
221	

222	
	CC002253187A
223	
224	
	DC001575649A
225	
226	
	CC009162808A
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228	
	DC020577063A
229	
230	
	CC026355372A
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234	
	CC003923767A
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236	
	CC398942280A
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	DC039880242A
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240	
	DC003620183A
241	
242	
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243	
244	
	DC010315550A
245	

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246
247 CC022829673A
248
249 DC028953121A
250
251 CC017876535A
252
253 DC004200587A
254
255 DC57721566A
256
257 CC42278420A
258
259 CC23069756A
260
261 CC03488590A
262
263 CC00262698A
264
265 CC00010750A
266
267 CC000000740A
268
269 C1CA

270
271 CC15443144A
272
273 DC67278579A
274
275 DC18156897A
276
277 DC01919402A
278
279 DC00110404A
280
281 DC00004686A
282
283 C1C25331414A
284
285 DC07832358A
286
287 CC02189568A
288
289 DC01062446A
290
291 CC00587872A
292
293 DC00251540A

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294
CC00053208A
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296
C1C25331414A
297
298
CC23498619A
299
300
DC03655620A
301
302
CC01504268A
303
304
DC00780353A
305
306
CC00325614A
307
308
DC00068245A
309

1844 E 3
1024 A 00 fjern skalafaktorer
310 EE 3

310
C3D75A
311
312
C3D75000003A
313