Mark I
Time-Sharing Service

Information Systems

Time-Sharing Service

Program Library Users Guide

Laplace Transform Inversion

GENERAL ELECTRIC

INFORMATION SERVICE DEPARTMENT

ØK DATA

DelData

Laplace Transform Inversion

PROGRAM NAME

TRANS$

January 1968
Reprinted January 1969

INFORMATION SYSTEMS
GENERAL ELECTRIC

PREFACE

This users guide explains how the response in the time domain of a linear system can be computed by using the Mark I Time-Sharing Service of General Electric's Information Service Department.

This guide was originally prepared in January 1968. The April 1968 reprint incorporated a limitation change decreasing the number of transform equations which can be processed in one run from twelve to eleven. Both the April 1968 reprint and the January 1969 reprint include minor additions which further define program operation. This printing does not render the original and copies of the previous versions obsolete.

Users need not be programmers. However, familiarity with the system is required. Time-Sharing System Manual (229116) provides such information, and the manual should be used in conjunction with this one. For ease of comprehension, this guide includes a sample problem and its solution.

Laplace Transform Inversion is one in an ever-expanding library of time-sharing programs for use by subscribers to the Mark I Time-Sharing Service. Each program is designated by its six-character name (TRANS$, in this case) to store and retrieve it on the system. Library programs are classified as follows:

On-Line

On-line library programs can be accessed from any terminal that is connected to the system. The criteria for placing a library program on-line are its general utility and frequency of use. Unless classified as run-only (see below), on-line programs can be listed, modified, copied, or run at the discretion of the user.

To retrieve a program from the on-line library, its six-character name is used, followed by three asterisks. The three-asterisk suffix is not an integral part of the program name; it is only a requirement for retrieving the program.

An index to listings of programs in the on-line library is available by listing the library program CATLOG.

Off-Line

The off-line library consists of programs not in general demand. An index to the listings of off-line programs can be obtained by listing the library program CATOFF. Off-line library programs are available for direct placement in a specific user catalog on request from your General Electric representative.

Run-Only

Run-only is a term applied to programs that cannot be listed or permanently modified by the user. It is possible to have run-only programs in either the on-line or the off-line library. Run-only programs are designated by a dollar sign ($) in the sixth character position of the program name.

The terms under which library programs are made available to subscribers may vary between programs, or they may vary with a given program from time to time. General Electric reserves the right to change these terms at its discretion. Any questions regarding use of library programs should be directed to your General Electric representative.

1. INTRODUCTION

This BASIC program computes the response in the time domain of a linear system whose output is expressed as a Laplace transform in the ratio of two polynomials.

A linear system may be represented by differential equations which can be transformed out of the time domain by using the Laplace transform. By solving these transform equations for the desired output, the system response can be expressed as the ratio of two polynomials in the complex variable *s*. TRANS$*** can then be used to compute the response in the time domain from this system response equation.

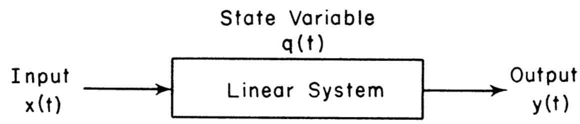
The option is available for plotting the output of the program versus time for any interval as specified by the user. On-line modifications can also be made for the time interval.

METHOD

The program computes the response in the time domain of a linear system whose desired output has been expressed as a Laplace transform in the ratio of two polynomials.

The recursive formula $q\left[(n+1)t\right] = e^{At}q(nt)$ is used by the program to compute the response in the time domain of the Laplace transform equation. This particular method was presented by M. L. Liou[1].

In the recursive formula, e^{At} is called a fundamental matrix, where A is a matrix consisting of constants and q is a column vector consisting of state variables (auxiliary dynamic variables). The derivation of this recursive formula starts with the general differential equations governing the behavior of a fixed linear continuous-time system.



The differential equations are:

$$\dot{q}(t) = Aq(t) + Bx(t)$$

$$y(t) = Cq(t) + Dx(t)$$

where A, B, C, and D are constant matrices and q(t), x(t), and y(t) are column vectors.

The solution for the continuous-time system is then derived, and from this the solution to the discrete-time system is computed to obtain the recursive formula. For a more detailed discussion, refer to the method as presented by M. L. Liou.

LIMITATIONS

The program can process up to eleven transform equations in one run.

The order of the denominator of the output transform equation must not exceed eleven and the order of the numerator must be less than the order of the denominator.

If more than one transform equation is to be processed, the equation with the highest order denominator polynomial must be entered first in the data lines and each succeeding equation entered in decreasing order of the denominator polynomial.

The maximum number of output data points that can be printed for each transform equation is 200.

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[1] Liou, M. L., *"A NOVEL METHOD OF EVALUATING TRANSIENT RESPONSE,"* Proceedings of the IEEE, Vol. 54, No. 1, January, 1966, pp. 20-23.

2. OPERATING INSTRUCTIONS

This program evaluates the transient response of a linear system shown below.

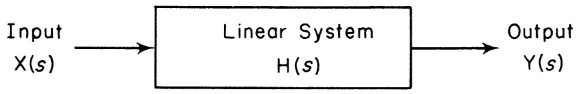

Figure 1.

First, express the desired output equation as a Laplace transform in the ratio of two polynomials, as follows:

$$Y(s) = H(s)\,X(s) = \frac{N(s)}{D(s)} \quad (1)$$

where

$$N(s) = F_n s^{(n-1)} + F_{(n-1)} s^{(n-2)} + \ldots F_1 \quad (2)$$

$$D(s) = s^n + G_n s^{(n-1)} + G_{(n-1)} s^{(n-2)} + \ldots G_1 \quad (3)$$

Note that the order of the numerator $N(s)$ must be less than the order of the denominator $D(s)$.

To use the program, perform the following calling sequence supplying all underlined information.

SYSTEM – BASIC

NEW ØR ØLD – ØLD

ØLD FILE NAME – TRANS$***

READY

Enter data beginning at line 500. In entering data, consecutive cases must be entered in descending order beginning with the case with the highest order denominator polynomial and ending with the case having the lowest order denominator polynomial. Also, all zero coefficients must be entered into the data line. The order in which the data is entered is shown below:

500 DATA CASE

510 DATA N1

520 DATA $F_{1,1}, F_{1,2}, \ldots\ldots\ldots\ldots F_{1,N1}$

530 DATA $G_{1,1}, G_{1,2}, \ldots\ldots\ldots\ldots G_{1,N1}$

540 DATA N2

550 DATA $F_{2,1}, F_{2,2}, \ldots\ldots\ldots\ldots F_{2,N2}$

560 DATA $G_{2,1}, G_{2,2}, \ldots\ldots\ldots\ldots G_{2,N2}$

.
.
.
.

through

.
.
.

9998

where,

CASE = number of transform equations to be processed.

N1 = order of denominator of response equation for CASE #1.

$F_{1,N1}$ = coefficients of numerator of response equation for CASE #1 (see equation 2).

$G_{1,N1}$ = coefficients of denominator of response equation for CASE #1 (see equation 3).

N2 = order of denominator of response equation for CASE #2.

$F_{2,N2}$ = coefficients of the numerator of response equation for CASE #2.

$G_{2,N2}$ = coefficients of the denominator of response equation for CASE #2.

The data for each transform equation to be analyzed is entered in the same manner as indicated above for CASE #1 and CASE #2. A maximum of eleven consecutive transform equations can be processed by the program.

Elements in the fundamental matrix are computed within an accuracy of 4 significant digits. However, the accuracy can be altered, as follows, by changing the number of significant digits (d) in line 22 of the program before the run.

Type 22 LET D=d

where d = number of significant digits of accuracy required.

After the data has been entered, type RUN followed by a carriage return. The words TIME INTERVAL (START, END, INCREMENT) = ? will appear at the teletypewriter. Supply this information followed by a carriage return and the response will be printed. The start of the interval must be zero or an integral multiple of the increment.

The program will then request any change in the time interval and calculate the response for a new time interval, if desired. If another time interval is not desired, the program will provide the option of obtaining a plot of the response. The next case will then be analyzed, repeating the same procedure, as above. A message will advise the user if an exceptionally long time is required to converge to a solution. If overflow or underflow occurs, this is usually an indication that the endpoint of the time interval is too large.

3. SAMPLE PROBLEMS

Problem #1:

In Figure 2, the switch is closed at time(t) = 0 with zero inductor current at t = 0-.

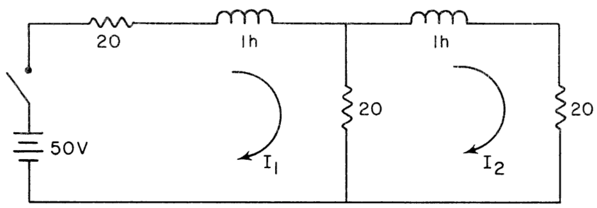

Figure 2.

The transform equations may be written in the following manner:

$$(s + 40)\, I_1(s) - 20 I_2(s) = 50/s$$

$$-20 I_1(s) + (s + 40)\, I_2(s) = 0$$

Therefore:

$$I_2(s) = \frac{\begin{vmatrix} s+40 & 50/s \\ -20 & 0 \end{vmatrix}}{\begin{vmatrix} s+40 & -20 \\ -20 & s+40 \end{vmatrix}} = \frac{1000}{s(s^2 + 80s + 1200)}$$

$$I_2(s) = \frac{1000}{s^3 + 80s^2 + 1200s}$$

With the output transform equation in this form, data can now be entered into the program.

Problem #2:

The transient response is desired of the following transform equation:

$$I = \frac{-4s^3 + 60s^2 - 360s + 840}{(s^5 + 16s^4 + 120s^3 + 480s^2 + 840s)}$$

Data for these two transform equations is entered as shown below by supplying all underlined information. The solution for each problem is shown as it appears at the teletypewriter.

INPUT FOR SAMPLE PROBLEMS

In entering data for Problems #1 and #2, Problem #2 must be entered first since it contains the denominator polynomial of the highest order. Therefore, Problem #2 is the first data case to be analyzed after which the program analyzes Problem #1.

```
OLD
OLD FILE NAME--TRANS$***

READY.


500 DATA 2
505 DATA 5
510 DATA 840,-360,60,-4,0
520 DATA 0,840,480,120,16
530 DATA 3
540 DATA 1000,0,0
550 DATA 0,1200,80
RUN
```

In line 500, the number of transform equations to be analyzed (2) is entered.

In line 505, the order of the denominator (5) of the transform equation in Problem #2 is entered.

In line 510, the coefficients in the numerator of the transform equation in Problem #2 are entered. Note that 0 is entered as the coefficient for the s^4 term.

In line 520, the coefficients in the denominator of the transform equation in Problem #2 are entered. Note that a zero is entered for the constant term which is missing from the denominator.

In line 530, the order of the denominator (3) of the transform equation in Problem #1 is entered.

In line 540, the coefficients in the numerator of the transform equation in Problem #1 are entered. Note that a 0 coefficient is entered for both the s^2 and s terms.

In line 550, the coefficients in the denominator of the transform equation in Problem #1 are entered. Note that a 0 is entered for the constant term which is missing from the denominator.

OUTPUT FOR SAMPLE PROBLEMS

```
TRANS$

CASE 1

TIME INTERVAL [START,END,INCREMENT]=? 0,2.5,.1

TIME           ØUTPUT

 0              0
 .1            -2.58523 E-2
 .2             .108951
 .3             9.03538 E-2
 .4            -3.53625 E-2
 .5            -.146287
 .6            -.16073
 .7            -6.05279 E-2
 .8             .12624
 .9             .352241
 1.             .572563
 1.1            .756448
 1.2            .890099
 1.3            .973764
 1.4            1.01644
 1.5            1.03062
 1.6            1.02837
 1.7            1.01913
 1.8            1.00907
 1.9            1.00131
 2.             .996786
 2.1            .995138
 2.2            .995432
 2.3            .996692
 2.4            .998163
 2.5            .999393

NEW TIME INTERVAL [YES=1,NØ=0]? 0

DØ YØU WANT A PLØT ØF THE ØUTPUT [YES=1,NØ=0]? 1
```

```
FØR X:    TØP =  0      BØTTØM =  2.5        INCREMENT =  .1
FØR Y:  LEFT = -.16073    RIGHT =  1.03062   INCREMENT =  1.98559 E-2

  I.........I.........I.........I.........I.........I.........I
-         +
.        +
.               +
.              +
.       +
.  +
. +
.      +
.               +
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-                                                           +
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.                                                           +
.                                                           +
.                                                           +
```

CASE 2

TIME INTERVAL [START, END, INCREMENT]=? 0, 1.9, .1

TIME	ØUTPUT
0	0
.1	.665197
.2	.810441
.3	.830235
.4	.832914
.5	.833277
.6	.833326
.7	.833332
.8	.833333
.9	.833333
1.	.833333
1.1	.833333
1.2	.833333
1.3	.833333
1.4	.833333
1.5	.833333
1.6	.833333
1.7	.833333
1.8	.833333
1.9	.833333

```
NEW TIME INTERVAL [YES=1,NØ=0]? 1

TIME INTERVAL [START,END,INCREMENT]=? 0,.4,.05

TIME            ØUTPUT

 0               0
 .05             .394229
 .1              .665197
 .15             .771151
 .2              .810441
 .25             .824911
 .3              .830235
 .35             .832193
 .4              .832914

NEW TIME INTERVAL [YES=1,NØ=0]? 0

DØ YØU WANT A PLØT ØF THE ØUTPUT [YES=1,NØ=0]? 0
```

Computer Centers and offices of the Information Service Department are located in principal cities throughout the United States.

Check your local telephone directory for the address and telephone number of the office nearest you. Or write . . .

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