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SCANDINAVIAN INFORMATION PROCESSING SYSTEMS

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fft

KEY-WORDS: RC 4000, Software, fft, Algol Procedure, ISO Tape

ABSTRACT: The procedure fft calculates the Fourier sum

$Y(s) = \mathrm{Sum}(X(t)\exp(\pm 2\pi i \times t \times s/N))$, $N = 2^m$, $t = 0, 1, \ldots, N-1$

for $s = 0, 1, \ldots, N-1$ by means of the Cooley-Tukey algorithm (the 'Fast Fourier Transform').

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procedure fft(A, B, m, sg);

1. Function and parameters.

Call parameters:

m	integer value. m determines the dimension N = 2××m of A and B. m must be < 24 to be meaningful, but the largest possible value is m = 13 in a computer with 128 k bytes storage capacity (corresponding to 32 k reals).
sg	integer value. sg = +1 gives + sign in the sum sg = -1 gives - sign in the sum.

Call and Return parameters:

A, B(0:N-1)	real arrays. They must on entry contain the real and imaginary part of X(t) in normal order. Upon exit they contain the real and imaginary part of the Fourier sum Y(s), also in normal order.

2. Method.

The procedure fft calculates the Fourier sum

$$Y(s) = \sum_{t} X(t)\exp(\pm 2\pi i \times t \times s/N)),\ N = 2\times\times m,\ t = 0, 1, \ldots, N-1$$

for s = 0, 1, ..., N-1 by means of the Cooley-Tukey algorithm (the 'Fast Fourier Transform').

The first part of the procedure (p:= 0; --- shift (m-j-24) end;) delivers the data A, B in reverse binary order. The second part (p:= 1; ... p:= p1 end) performs the summation in place. So it is possible to carry out the summation with the data A, B in reverse binary order simply by omitting the first part.

The loss of accuracy is almost proportional to m and is for m = 8 about 1 significant decimal.

The running time is proportional to N×m and is for m = 13 about 45 seconds.

3. References.

About the Cooley-Tukey algorithm and its implementation, see:

[1] Cooley, J.W., and Tukey, J.W.: An algorithm for the machine calculation of complex Fourier series. Math. Comp. 19, 90 (April 1965), p. 297-301.

[2] R.C. Singleton: On computing the fast Fourier Transform. Communications of the ACM, Vol. 10, N. 10, (October 1967), p. 647-654.

4. Algol procedure.

```
fft=set 2
frt=algol index.no message.yes

external
procedure fft (A,B,m,sg);
message fft version 20.10.69;
value m,sg; integer m,sg; array A,B;
begin integer i,j,k,n,p,p1,q,j1,j2;
  real r,x0,x1,y0,y1,c,s,c1,C,S;
  n:=1 shift m-1;
  r:=6.28318530718; p:=0;
  for i:=0 step 1 until n do
  begin
    if i<p then
    begin
      c:=A(i); A(i):=A(p); A(p):=c;
      c:=B(i); B(i):=B(p); B(p):=c
    end;
    k:=p shift (24-m); j:=-1;
    for j:=j+1 while k<0 do k:= k shift 1;
    p:= (-8388607-1+k) shift (m-j-24)
  end;
  p:=1;
  for i:=1 step 1 until m do
  begin
```

```
      r:=r/2; S:=sg×sin(r); C:=2×sin(r/2)××2;
      c:=1; s:=0; q:=p-1; p1:=2×p;
      for j:=0 step 1 until q do
      begin
        for k:=0 step p1 until n do
        begin
          j1:=j+k; j2:=j1+p;
          x0:=A(j2); y0:=B(j2);
          x1:=x0×c-y0×s; y1:=x0×s+y0×c;
          x0:=A(j1); y0:=B(j1);
          A(j1):=x0+x1; B(j1):=y0+y1;
          A(j2):=x0-x1; B(j2):=y0-y1
        end;
        c1:=c; c:=c-(C×c+S×s); s:=s+(S×c1-C×s)
      end;
      p:=p1
    end
  end;
end
```