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A/S REGNECENTRALEN
SCANDINAVIAN INFORMATION PROCESSING SYSTEMS

SYSTEM LIBRARY

RCSL NO: 53-M1
TYPE : Algol 5 Procedure
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EDITION: January 1970 (E)

RC 4000 SOFTWARE

MATHEMATICAL PROCEDURE LIBRARY

zero1(x, F, a, b, eps)

ABSTRACT

The boolean procedure zero1 evaluates a zero of an arbitrary real function. The method is an adaptive method based on regula falsi and bisection.

1. Function and Parameters.

1: Function:

The boolean procedure evaluates one zero of the function F(x) within the interval a <= x <= b. The method is based on regula falsi and bisection combined with an adaptive parameter giving the weights of regula falsi and bisection.

Call parameters:

a, b: real value parameters specifying the end points of the interval within which the zero is calculated. This interval is a <= x <= b if a < b, otherwise a <= x <= b.

eps: A real name parameter giving the accuracy with which the zero is determined.

Relative accuracy may be specified by substituting an expression like delta × x for eps.

If eps specifies an accuracy that is not obtainable calculations are stopped with the obtainable accuracy.

Return parameters:

x: a real name parameter being the independent variable in the expression giving F.

On exit the z e r o determined by zero1.

Need not be initialized.

zero1: The boolean procedure name is set to false if $F(a) > 0$ and $F(b) > 0$ or $F(a) < 0$ and $F(b) < 0$, otherwise zero1 is true.

Other parameters:

F: a real name parameter specifying the function for which the zero is to be evaluated.
F m u s t be supplied as an expression depending on x.

2. Method

The procedure calculates for each iteration a new value as a weighted mean between a regula falsi and a bisection value:

$a \leq x \leq b$ being the interval in which the zero is to be evaluated, with $fa = F(a) > 0$ and $fb = F(b) < 0$, the following algorithm is used:

$xr = a - fa \times (b - a) / (fb - fa)$
(i.e. x value obtained by regula falsi)

$xb = (b + a) / 2$
(i.e. x value obtained by bisection).

The new value of x is now calculated as

$x = xr + (xb - xr) \times vb$

where the weight factor, vb satisfies $0 \leq vb \leq 1$.

And the value of vb is calculated as

```
vb := if a < xr and xr < b then vb × vb / 2 else 1;
```

i.e. if xr, the x value calculated by regula falsi method, is inside the new interval then regula falsi might be better than the x just calculated and more weight are given to regula falsi in the next iteration (i.e. smaller vb), otherwise the next iteration is pure bisection (vb = 1).

f = sg × F(x) is evaluated for the new x value and a new interval (a|b) is determined as:

```
if f > 0 then begin b := x; fb := f end
         else begin b := x; fa := f end;
```

The factor sg is f = sg × F(x) is introduced in order to give a simple algorithm inside the iteration loop.

Before starting iteration sg is initialized as

```
sg := id fa > 0 then 1 else -1;
```

and all values of F are multiplied by sg, (i.e. fa > 0 and fb < 0).

If the parameters specifying a and b gives b < a then a interchange of these two parameters are made in the start of the program.

However if F(b) and F(a) are both either greater than or less than 0 then the method does not work and the boolean name zero1 is set to false indicating that no zero is evaluated, otherwise zero1 is true.

3. Accuracy and storage requirement.

3.1. The accuracy is determined by the input parameter eps giving the absolute precision of the zero. If however an expression giving eps includes the factor x (the independent variable) then relative precision is automatically used.

If an accuracy higher, than the one obtainable in RC 4000, is specified then a result with the highest obtainable precision is delivered.

3.2. Storage requirements:

1 segment + 9 real variables

4. Test and discussion.

zero1 is tested by use of the 6 functions used in ref. 1 for test of Gier procedures.

Results of this test using testprogram as given in section 7 are:

Textexamples for : external boolean procedure zero1(x,F,a,b,eps)

F(x)	a	b	eps	x	iter
5.33+2.6×x	-9.9	2.1	'-6	-2.05' +0	8
ln(x/0.7)	0.1	2	'-8	7.00' -1	12
exp(x)-0.4	-5	1	x×'-7	-9.16' -1	12
sin(x)-sin(1.55)	-3	1.59	'-5	1.59' +0	11
x××3 + x	-0.5	2	'-8+abs(x)×'-6	1.93'-15	9
x××5	-1	2	'-6	6.96' -7	24

x = the zero calculated by zero1
iter = the number of references to F

These result may be compared with results from ref. 1 showing that although using a very simple strategy zero1 is very fast.

5. References.

Bo Munch-Andersen: Zero, Algol procedure, Regnecentralen October 1965, Gier System Library, Order No. 409.

6. Algol program

```
zero1=set 1
zero1=algol
external


boolean procedure zero1(x,F,a,b,eps);
value a,b; real x,F,a,b,eps;
begin
  real fa,fb,f,vb,sg,v,xr;
   comment 1;

    zero1:= true;
    if a > b then begin f := a; a := b; b := f end;
    x  := a;
    f  := F;
    sg := if f > 0 then 1 else -1;
    fa := sg×f;
    if fa = 0 then goto out;
    x  := b;
    fb := sg×F;
    if fb = 0 then goto out;
    if fb > 0 then begin zero1 := false; goto out end;
    vb := 1;

next:
    v  := b-a;
    x  := (b+a)/2;
    if v < 2×abs(eps)  or  v < 1.2₁₀-10×abs(x) then goto out;
    comment 2;
    xr := a-fa×v/(fb-fa);
    x  := xr+(x-xr)×vb;
    f  := sg×F;
    if f = 0 then goto out else
    if f > 0 then begin a := x; fa := f end
             else begin b := x; fb := f end;
    comment 3;
    vb := if a < xr  and  xr < b then vb×vb/2 else 1;
    goto next;

out:
end;

comment

1:
   Reference:

         RC4000 System Library
         Order No. 55-D44
         A/S Regnecentralen, July 1969
         N. Schreiner Andersen
```

Function:

The boolean procedure evaluates one zero of the function F(x) within the interval a <= x <=b. The method is based on regula falsi and bisection combined with an adaptive parameter giving the weights of regula falsi and bisection.

Call parameters:

a,b	:	real value parameters specifying the end points of the interval within which the zero is calculated. This interval is a <= x <= b if a < b otherwise b <= x <= a.
eps:		A real name parameter giving the accuracy for which the zero is determined. Relative accuracy is specified through an expression with factor x, i.e. x×₁₀-7 gives a relative accuracy of ₁₀-7. If eps specifies an accuracy that is not obtainable within RC4000 calculations are stopped with the obtainable accuracy.

Return parameters:

x :	a real name parameter being the independent variable in the expression giving F. On exit the z e r o determined by zero1.
zero1 :	The boolean procedure name is set to false if F(a) > 0 and F(b) > 0 or F(a) < 0 and F(b) < 0 , otherwise zero1 is true.

Other parameters:

F :	a real name parameter specifying the function for which the zero is to be evaluated. F m u s t be supplied with an expression depending on x.

2: In order to avoid that calculations can not stop because of too small eps (below the precision obtainable on RC4000) a security is put in here causing stop on v < 1.2₁₀-10×abs(x).

3: A new weight, vb is calculated before next iteration;

end zero1;

7. Testprogram

```
A/S Regnecentralen
Testprogram for procedure zerol
NSA, 1.09.69.

begin
real procedure F(n);
integer n;
 begin
  i := i + 1;
  F := case n of (5.33+2.6××x,ln(x/0.7),exp(x)-0.4,sin(x)-sin(1.55),
                  x××3 + x, x××5);
 end F;

real x; integer i;

write(out,<:
Testexamples for : external boolean procedure zerol(x,F,a,b,eps)

:>);
write(out,<:
    F(x)              a      b      eps                  x         iter
:>);

i := 0; zero1(x,F(1),-9.9,2.1,₁₀-6);
write(out,<:
5.33+2.6××x         -9.9  2.1     ₁₀-6            :>,<<   -d.dd₁₀+dd>,x,<<   -dd>,i);

i := 0; zero1(x,F(2),0.1,2,₁₀-8);
write(out,<:
ln(x/0.7)            0.1  2       ₁₀-8            :>,<<   -d.dd₁₀+dd>,x,<<   -dd>,i);

i := 0; zero1(x,F(3),-5,1,x×₁₀-7);
write(out,<:
exp(x)-0.4          -5    1       x×₁₀-7          :>,<<   -d.dd₁₀+dd>,x,<<   -dd>,i);

i := 0; zero1(x,F(4),3,1.59,₁₀-5);
write(out,<:
sin(x)-sin(1.55)    -3    1.59    ₁₀-5            :>,<<   -d.dd₁₀+dd>,x,<<   -dd>,i);

i := 0; zero1(x,F(5),-0.5,2,₁₀-8+abs(x)×₁₀-6);
write(out,<:
x××3 + x            -0.5  2       ₁₀-8+abs(x)×₁₀-6:>,<<   -d.dd₁₀+dd>,x,<<   -dd>,i);

i := 0; zero1(x,F(6),-1,2,₁₀-6);
write(out,<:
x××5                -1    2       ₁₀-6            :>,<<   -d.dd₁₀+dd>,x,<<   -dd>,i);

write(out,<:<10><10><10>
            x    =  the zero calculated by zero1
            iter =  the number of references to F
:>);
end testprogram;
```