Application Program

H20-0166-2

# System/360 Scientific Subroutine Package

# (360A-CM-03X)

## Application Description

Part I describes the current Scientific Subroutine Package (SSP), a collection of FORTRAN subroutines divided, for the sake of presentation, into three groups: statistics, matrix manipulation, and other mathematics. Part I includes brief abstracts of the current subroutines and some of their common characteristics.

Part II describes Version 2, which is not yet available but will be available in the near future. Version 2 will be a major extension including over 70 additional mathematical subroutines. Part II includes brief abstracts of the Version 2 subroutines and some of their common characteristics.

CONTENTS

PART I



PART II



This edition, H20-0166-2, is a major revision of, and obsoletes, H20-0166-1. Version 2, described in Part II, is not yet available but will be available in the near future.

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# PART I

# INTRODUCTION

## OVERVIEW

The Scientific Subroutine Package (SSP) is a collection of FORTRAN subroutines divided, for the sake of presentation, into three groups: statistics, matrix manipulation, and other mathematics. It is a collection of input/output-free computational building blocks that can be combined with a user's input, output, or computational routines to meet his needs. The package can be applied to the solution of many problems in industry, science, and engineering.

The primary purpose and objective of the Scientific Subroutine Package is to make available a mathematical and statistical subroutine library. The user may supplement or modify the collection to meet his needs. This library includes a wide variety of subroutines to perform the functions listed below, but is not intended to be exhaustive in terms of either functions performed or methods used.

## CONTENT

Individual subroutines, or a combination of them, can be used to carry out the listed functions in the following areas:

### Statistics

- Analysis of variance (factorial design)
- Correlation analysis
- Multiple linear regression
- Polynomial regression
- Canonical correlation
- Factor analysis (principal components, varimax)
- Discriminant analysis (many groups)
- Time series analysis
- Data screening and analysis
- Nonparametric tests
- Random number generation (uniform, normal)

### Matrix Manipulation

- Inversion
- Eigenvalues and vectors (real symmetric case)
- Simultaneous linear algebraic equations
- Transpositions
- Matrix arithmetic (addition, product, etc.)
- Partitioning
- Tabulation and sorting of rows or columns
- Elementary operations on rows or columns

### Other Mathematical Areas

- Integration of given or tabulated functions
- Integration of up to six first-order differential equations
- Fourier analysis of given or tabulated functions
- Bessel and modified Bessel function evaluation
- Gamma function evaluation
- Legendre function evaluation
- Elliptic, exponential, sine, cosine, Fresnel integrals
- Finding real roots of a given function
- Finding real and complex roots of a real polynomial
- Polynomial arithmetic (addition, division, etc.)
- Polynomial evaluation, integration, differentiation

## CHARACTERISTICS

Some of the characteristics of the Scientific Subroutine Package are:

- All subroutines are free of input/output statements
- Subroutines do not contain fixed maximum dimensions for the data arrays named in their calling sequences
- All subroutines are written in FORTRAN
- Many matrix manipulation subroutines handle symmetric and diagonal matrices (stored in economical, compressed formats) as well as general matrices. This can result in considerable saving in data storage for large arrays.
- The use of the more complex subroutines (or groups of them) is illustrated in the program documentation by sample main programs with input/output
- All subroutines are documented uniformly

## DESIGN PHILOSOPHY

### Choice of Algorithms

The algorithms in SSP have been chosen after considering questions of storage, accuracy, and past experience with the algorithm. Conservation of storage has been the primary criterion except in those situations where other considerations outweighed that of storage. As a result, many compromises have been made both with respect to level of sophistication and execution time. One such compromise is the use of the Runge-Kutta integration technique rather than predictor-corrector methods. A departure from the primary criterion of storage is illustrated by the algorithm for matrix inversion. If only row pivoting had been used, the subroutine would not have required working storage and would have needed fewer FORTRAN statements for implementation. However, pivoting on both rows and columns was chosen because of the accuracy requirement for matrix inversion in statistical operations.

## Programming

The subroutines in SSP have been programmed in the subset of the FORTRAN IV language which is compatible with all of the FORTRAN processors announced for System/360. Many of the larger functions such as those in statistics have been programmed as a series or sequence of subroutines.

An example of the use of sequences of subroutines is the statistical function called factor analysis. Factor analysis is a method of analyzing the underlying relations among a set of variables. It determines whether the variance in the original set of variables can be accounted for adequately by a smaller number of basic categories; namely, factors. In the Scientific Subroutine Package, factor analysis is normally performed by calling the following five subroutines in sequence:

1. CORRE - to find means, standard deviations, and correlation matrix
2. EIGEN - to compute eigenvalues and associated eigenvectors of the correlation matrix
3. TRACE - to select the eigenvalues that are greater than or equal to the control value specified by the user
4. LOAD - to compute a factor matrix
5. VARMX - to perform varimax rotation of the factor matrix

The multiple use of subroutines is illustrated by the fact that subroutine CORRE is also utilized in the multiple linear regression and canonical correlation. Subroutine EIGEN is used in canonical correlation as a third level subroutine.

# SUBROUTINES

## GENERAL REMARKS

Below are listed the subroutines of SSP/360, grouped into related functional areas. In the case of six statistical entries, multiple linear regression to factor analysis, the abstract gives the sequence of several SSP subroutines needed to perform the statistical function.

A tabulation of the subroutines of SSP, with detailed characteristics, is given in the Appendices.

## STATISTICS

### Data Screening

TALLY — totals, means, standard deviations, minimums, and maximums

BOUND — selection of observations within bounds

SUBST — subset selection from observation matrix

ABSNT — detection of missing data

TAB1 — tabulation of data (1 variable)

TAB2 — tabulation of data (2 variables)

SUBMX — build subset matrix

### Elementary Statistics

MOMEN — first four moments

TTEST — tests on population means

### Correlation

CORRE — means, standard deviations, and correlations

### Multiple Linear Regression

Abstract (CORRE, ORDER, MINV, MULTR in sequence)

ORDER — rearrangement of intercorrelations

MULTR — multiple regression and correlation

### Polynomial Regression

Abstract (GDATA, ORDER, MINV, MULTR in sequence)

GDATA — data generation

### Canonical Correlation

Abstract (CORRE, CANOR, MINV, NROOT, EIGEN in sequence)

CANOR — canonical correlation

NROOT — eigenvalues and eigenvectors of a special nonsymmetric matrix

### Analysis of Variance

Abstract (AVDAT, AVCAL, MEANQ in sequence)

AVDAT — data storage allocation

AVCAL — $\Sigma$ and $\Delta$ operation

MEANQ — mean square operation

### Discriminant Analysis

Abstract (DMATX, MINV, DISCR in sequence)

DMATX — means and dispersion matrix

DISCR — discriminant functions

### Factor Analysis

Abstract (CORRE, EIGEN, TRACE, LOAD, VARMX in sequence)

TRACE — cumulative percentage of eigenvalues

LOAD — factor loading

VARMX — varimax rotation

### Time Series

AUTO — autocovariances

CROSS — crosscovariances

SMO — application of filter coefficients (weights)

EXSMO — triple exponential smoothing

Nonparametric Statistics

CHISQ — $\chi^2$ test for a contingency table

UTEST — Mann-Whitney U-test

TWOAV — Friedman two-way analysis of variance

QTEST — Cochran Q-test

SRANK — Spearman rank correlation

KRANK — Kendall rank correlation

WTEST — Kendall coefficient of concordance

RANK — rank observations

TIE — calculation of ties in ranked observations

Random Number Generators

RANDU — uniform random numbers

GAUSS — normal random numbers

MATRIX MANIPULATION

MINV — matrix inversion

EIGEN — eigenvalues and eigenvectors of a real, symmetric matrix

SIMQ — solution of simultaneous linear, algebraic equations

GMADD — add two general matrices

GMSUB — subtract two general matrices

GMPRD — product of two general matrices

GMTRA — transpose of a general matrix

GTPRD — transpose product of two general matrices

MADD — add two matrices

MSUB — subtract two matrices

MPRD — matrix product (row into column)

MTRA — transpose a matrix

TPRD — transpose product

MATA — transpose product of matrix by itself

SADD — add scalar to matrix

SSUB — subtract scalar from a matrix

SMPY — matrix multiplied by a scalar

SDIV — matrix divided by a scalar

RADD — add row of one matrix to row of another matrix

CADD — add column of one matrix to column of another matrix

SRMA — scalar multiply row and add to another row

SCMA — scalar multiply column and add to another column

RINT — interchange two rows

CINT — interchange two columns

RSUM — sum the rows of a matrix

CSUM — sum the columns of a matrix

RTAB — tabulate the rows of a matrix

CTAB — tabulate the columns of a matrix

RSRT — sort matrix rows

CSRT — sort matrix columns

RCUT — partition row-wise

CCUT — partition column-wise

RTIE — adjoin two matrices row-wise

CTIE — adjoin two matrices column-wise

MCPY — matrix copy

XCPY — copy submatrix from given matrix

RCPY — copy row of matrix into vector

CCPY — copy column of matrix into vector

DCPY — copy diagonal of matrix into vector

SCLA — matrix clear and add scalar

DCLA — replace diagonal with scalar

MSTR — storage conversion

MFUN — matrix transformation by a function

RECP — reciprocal function for MFUN

LOC — location in compressed-stored matrix

CONVT — single precision-double precision conversion

ARRAY — vector storage-double dimensioned storage conversion

OTHER MATHEMATICAL AREAS

Integration

QUADR — integral of tabulated function

SMPSN — integral of given function by Simpson's rule

RK1 — integral of first-order differential equation by Runge-Kutta method

RK2 — tabulated integral of first-order differential equation by Runge-Kutta method

RK3 — tabulated integral of a system of six first-order differential equations by Runge-Kutta method

Fourier Analysis

FORIF — Fourier analysis of a given function

FORIT — Fourier analysis of a tabulated function

Special Operations and Mathematical Functions

GAMMA — gamma function

LEGEN — Legendre polynomial

BESJ — J Bessel function

BESY — Y Bessel function

BESI — I Bessel function

BESK — K Bessel function

CELI1 — elliptic integral of the first kind

CELI2 — elliptic integral of the second kind

EXPI — exponential integral

SICI — sine, cosine integral

CS — Fresnel integrals

Roots of Nonlinear Equations

RTWIT — refine estimate of root by Wegstein's iteration

RTMIT — determine root within a range by Mueller's iteration

RTNIT — refine estimate of root by Newton's iteration

Roots of Polynomial

POLRT — real and complex roots of a real polynomial

Polynomial Operations

PADD — add two polynomials

PADDM — multiply polynomial by constant and add to another polynomial

PCLA — replace one polynomial by another

PSUB — subtract one polynomial from another

PMPY — multiply two polynomials

PDIV — divide one polynomial by another

PQSD — quadratic synthetic division of a polynomial

PVAL — value of a polynomial

PVSUB — substitute variable of polynomial by another polynomial

PCLD — complete linear synthetic division

PILD — evaluate polynomial and its first derivative

PDER — derivative of a polynomial

PINT — integral of a polynomial

PGCD — greatest common divisor of two polynomials

PNORM — normalize coefficient vector of polynomial

## OVERALL RULES OF USAGE

### SUBROUTINE USAGE

All subroutines in the Scientific Subroutine Package (SSP) are entered by means of the standard FORTRAN CALL statement. These subroutines are purely computational in nature and do not contain any references to input/output devices. The user must therefore furnish, as part of his program, whatever input/output and other operations are necessary for the total solution of his problem. In addition, the user must define by DIMENSION statements all matrices to be operated on by SSP subroutines as well as those matrices utilized in his program. The subroutines contained in SSP are no different from any user-supplied subroutine. All of the normal rules of FORTRAN concerning subroutines must, therefore, be adhered to with the exception that the dimensioned areas in the SSP subroutine are not required to be the same as those in the calling program.

### MATRIX OPERATIONS

Special consideration must be given to the subroutines that perform matrix operations. These subroutines have two characteristics that affect the format of the data in storage — variable dimensioning and data storage compression.

#### Variable Dimensioning

Those subroutines that deal with matrices can operate on any size array limited, in most cases, only by the available core storage and numerical analysis considerations. The subroutines do not contain fixed maximum dimensions for data arrays named in their calling sequence. The variable dimension capability has been implemented in SSP by using a vector storage approach. Under this approach, each column of a matrix is immediately followed in storage by the next column. Vector storage and two-dimensional storage result in the same layout of data in core, so long as the number of rows and columns in the matrix are the same as those in the user's dimension statement. If, however, the matrix is smaller than the dimensioned area, the two forms of storage are not compatible. A subroutine called ARRAY is available in SSP to change from one form of storage to the other. In addition, a subroutine called LOC is available to assist in referencing elements in an array stored in the vector fashion.

#### Storage Compression

Many subroutines in SSP can operate on compressed forms of matrices, as well as the normal form. Using this capability, which is called "storage mode", considerable savings in data storage can be obtained for special forms of large arrays. The three modes of storage are termed general, symmetric, and diagonal. In this context, general mode is one in which all elements of the matrix are in storage. Symmetric mode is one in which only the upper triangular portion of the matrix is retained columnwise in sequential locations in storage. (The assumption is made that the corresponding elements in the lower triangle have the same value.) Diagonal mode is one in which only the diagonal elements of the matrix are retained in sequential locations in storage. (The off-diagonal elements are assumed to be zero.) This capability has been implemented using the vector storage approach.

A special set of matrix subroutines is included in SSP. These subroutines (GMADD, GMSUB, GMPRD, GMTRA, and GTPRD) execute faster than their counterparts (MADD, MSUB, MPRD, MTRA, and TPRD) because they do not have the storage mode capability.

### SAMPLE PROGRAMS

Distributed with the subroutines of SSP are a dozen or so sample main programs with input/output, control (parameter) cards, and sample data. These sample main programs serve two purposes. First, they demonstrate input/output and the use of sequences of subroutines to carry out higher-level functions. Second, a user may attach his own data (in similar formats) to most of the main programs in order to accomplish the functions of the programs.

There are sample main programs to do each of the following operations (parentheses contain the code name of the main program):

1. Data screening (DASCR)
2. Regression (REGRE)
3. Polynomial regression (POLRG)

4. Canonical correlation (MCANO)
5. Analysis of variance, factorial design (ANOVA)
6. Discriminant analysis, many groups (MDISC)
7. Factor analysis (FACTO)
8. Exponential smoothing, third order (EXPON)
9. Matrix addition (ADSAM)
10. Integration of a tabulated function (QDINT)
11. Runge-Kutta solution of $dy/dx = f(x, y)$ (RKINT)
12. Polynomial roots (SMPRT)
13. Simultaneous linear equations (SOLN)

Below is the sample main program (QDINT) for numerical integration of a tabulated function, together with listings of input and output.

The execution time of this sample program on a System/360, Model 30, using an IBM 2540 Card Reader as input and an IBM 1403, Model 3 as output, in six seconds. The maximum number of tabulated values acceptable to the sample program may be changed by modifying the dimension statement in QDINT. The format of the parameter cards and data cards may be changed by modifying statements 10 and 32, respectively, in QDINT.

```
C
C     ...............................................................
C
C        SAMPLE PROGRAM FOR INTEGRATION OF A TABULATED FUNCTION BY
C        NUMERICAL QUADRATURE - QDINT
C
C        PURPOSE
C           INTEGRATES A SET OF TABULATED VALUES FOR F(X) GIVEN THE
C           NUMBER OF VALUES AND THEIR SPACING
C
C        REMARKS
C           THE NUMBER OF VALUES MUST BE MORE THAN ONE AND THE SPACING
C           GREATER THAN ZERO
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           QUADR
C
C        METHOD
C           READS CONTROL CARD CONTAINING THE CODE NUMBER, NUMBER OF
C           VALUES, AND THE SPACING OF THE FUNCTION VALUES CONTAINED
C           ON THE FOLLOWING DATA CARDS. DATA CARDS ARE THEN READ AND
C           INTEGRATION IS PERFORMED. MORE THAN ONE CONTROL CARD AND
C           CORRESPONDING DATA CAN BE INTEGRATED IN ONE RUN. EXECUTION
C           IS TERMINATED BY A BLANK CONTROL CARD.
C
C     ...............................................................
C
C        THE FOLLOWING DIMENSION MUST BE AS LARGE AS THE MAXIMUM NUMBER
C        OF TABULATED VALUES TO BE INTEGRATED
C
      DIMENSION Z(500)
C
C        ............................................................
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION Z,SPACE,ANS
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        ............................................................
C
   10 FORMAT (2I5,F10.0)
   20 FORMAT(1H1,64HINTEGRATION OF TABULATED VALUES FOR F(X) USING SUBR
     1OUTINE QUADR//1H ,10HFUNCTION  ,I5,3X,I5,17H TABULATED VALUES,
     25X,10HINTERVAL =,E15.8//)
   22 FORMAT(1H ,17HILLEGAL CONDITION/)
   23 FORMAT(1H ,43HNUMBER OF TABULATED VALUES IS LESS THAN TWO)
   24 FORMAT(1H ,26HSPECIFIED INTERVAL IS ZERO)
   30 FORMAT(1H ,7X,31HRESULTANT VALUE OF INTEGRAL IS ,E15.8)
   32 FORMAT(7F10.0)
C
   35 READ(5,10)ICOD,NUMBR,SPACE
      IF(ICOD+NUMBR)70,70,38
   38 WRITE(6,20)ICOD,NUMBR,SPACE
      IF(NUMBR-1)100,50,50
   50 READ(5,32)(Z(I),I=1,NUMBR)
      CALL QUADR(Z,NUMBR,SPACE,ANS,IER)
      IF(IER-1)60,100,200
   60 WRITE(6,30)ANS
      GO TO 35
   70 RETURN
  100 WRITE(6,22)
      WRITE (6,23)
      GO TO 35
  200 WRITE(6,22)
      WRITE(6,24)
      GO TO 35
      END
```

Input listing

```
/DATA
12345   20     1.0
       2.0       2.0       2.0       2.0       2.0       2.0       2.0
       2.0       2.0       2.0       2.0       2.0       2.0       2.0
       2.0       2.0       2.0       2.0       2.0       2.0       2.0
  543   10     1.0
       1.0       2.0       3.0       4.0       5.0       6.0       7.0
       8.0       9.0      10.0      11.0
```

Output listing

```
INTEGRATION OF TABULATED VALUES FOR F(X) USING SUBROUTINE QUADR

FUNCTION  12345      20 TABULATED VALUES     INTERVAL = 0.10000000E 01


       RESULTANT VALUE OF INTEGRAL IS  0.38000000E 02


INTEGRATION OF TABULATED VALUES FOR F(X) USING SUBROUTINE QUADR

FUNCTION    543      10 TABULATED VALUES     INTERVAL = 0.10000000E 01


       RESULTANT VALUE OF INTEGRAL IS  0.49500000E 02
```

## REQUIRED SYSTEMS

### MACHINE CONFIGURATION

A basic requirement is a System/360 configuration suitable for the FORTRAN compiler chosen. The machine configuration required for any given problem depends upon the number of subroutines used, the size of the compiled subroutines, the size of the compiled main program, the size of the control program, and the data storage requirements. Appendix A provides information needed to estimate memory requirements for each subroutine.

### PROGRAMMING SYSTEMS

System/360 Basic Programming Support FORTRAN Compiler (tape). However, the subroutines of SSP/360 will compile and execute with all currently announced System/360 FORTRAN compilers.

## PRECISION

The accuracy of the computations in many of the SSP subroutines is highly dependent upon the number of significant digits available for arithmetic operations. Matrix inversion, integration, and many of the statistical subroutines fall into this category. The user may, therefore, wish to have double precision versions of these subroutines.

The instructions for converting each of the critical subroutines to double precision are included as part of the subroutine description. FORTRAN double precision statements have been included in each of these subroutines in the form of a comments card. In most cases, the double precision version of the subroutine can be obtained merely by removing the C from column one of the double precision statement card(s) prior to compilation. In a few cases additional instructions are given for changing references to FORTRAN-furnished functions such as SQRT to DSQRT and ABS to DABS.

## REFERENCE MATERIAL

1. System/360 Scientific Subroutine Package (360A-CM-03X) Programmer's Manual (H20-0205-0). This manual contains sufficient information to permit the reader to understand and use all of the subroutines in the Scientific Subroutine Package.
2. System Manual. This manual contains flowcharts for all of the subroutines in the Scientific Subroutine Package.
3. IBM System/360 Basic Programming Support FORTRAN IV (C28-6504). This publication describes and illustrates the use of the Basic Programming Support FORTRAN IV Language for the IBM System/360.
4. IBM System/360 Basic Programming Support FORTRAN IV, 360 P-FO-031 Programmer's Guide (C28-6583). This publication describes the procedure for compiling and executing programs written in the Basic Programming Support FORTRAN IV Language.

## APPENDIX A: SUBROUTINES OF S/360 SSP (NAMES, DESCRIPTIONS, AND STORAGE REQUIREMENTS)

The following table lists the number of characters of storage required by each of the subroutines in the Scientific Subroutine Package. The figures given were obtained by using Basic Programming Support FORTRAN (Tape), Version 1, Level 0, June 1965. The use of other FORTRAN compilers on System/360 may cause deviations from these figures. The table also shows which subroutines or function subprograms are called or used by a given subroutine.

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| STATISTICS | | |
| Data Screening | | |
| TALLY | totals, means, standard deviations, minimums, maximums | 988 |
| BOUND | selection of observations within bounds | 668 |
| SUBST | subset selection from observation matrix (needs user's Boolean subroutines) | 888 |
| ABSNT | detection of missing data | 428 |
| TAB1 | tabulation of data (1 variable) | 1480 |
| TAB2 | tabulation of data (2 variables) | 2536 |
| SUBMX | build subset matrix | 508 |
| Elementary Statistics | | |
| MOMEN | first four moments | 1276 |
| TTEST | tests on population means | 1786 |
| Correlation | | |
| CORRE | means, standard deviations, and correlations (needs user's subroutine to get data) | 2844 |
| Multiple Linear Regression (usually requires CORRE, ORDER, MINV, MULTR in sequence) | | |
| ORDER | rearrangement of intercorrelations | 678 |
| MULTR | multiple regression and correlation | 1348 |
| Polynomial Regression (usually requires GDATA, ORDER, MINV, MULTR in sequence) | | |
| GDATA | data generation | 1742 |
| Canonical Correlation (usually requires CORRE, CANOR, MINV, NROOT, EIGEN in sequence) | | |
| CANOR | canonical correlation (CANOR calls MINV and NROOT) | 2824 |
| NROOT | eigenvalues and eigenvectors of a special nonsymmetric matrix (NROOT calls EIGEN) | 1936 |
| Analysis of Variance (usually requires AVDAT, AVCAL, MEANQ in sequence) | | |
| AVDAT | data storage allocation | 922 |
| AVCAL | $\Sigma$ and $\Delta$ operation | 794 |
| MEANQ | mean square operation | 1700 |
| Discriminant Analysis (usually requires DMATX, MINV, DISCR in sequence) | | |
| DMATX | means and dispersion matrix | 1170 |
| DISCR | discriminant functions | 2532 |
| Factor Analysis (usually requires CORRE, EIGEN, TRACE, LOAD, VARMX in sequence) | | |
| TRACE | cumulative percentage of eigenvalues | 560 |
| LOAD | factor loading | 486 |

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| VARMX | varimax rotation | 2864 |

Time Series

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| AUTO | autocovariances | 582 |
| CROSS | crosscovariances | 706 |
| SMO | application of filter coefficients (weights) | 538 |
| EXSMO | triple exponential smoothing | 616 |

Nonparametric Statistics

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| CHISQ | $\chi^2$ test for a contingency table | 1406 |
| UTEST | Mann-Whitney U-test (UTEST calls RANK and TIE) | 968 |
| TWOAV | Friedman two-way analysis of variance (TWOAV calls RANK) | 1100 |
| QTEST | Cochran Q-test | 710 |
| SRANK | Spearman rank correlation (SRANK calls RANK and TIE) | 1260 |
| KRANK | Kendall rank correlation (KRANK calls RANK and TIE) | 1598 |
| WTEST | Kendall coefficient of concordance (WTEST calls RANK and TIE) | 1488 |
| RANK | rank observations | 670 |
| TIE | calculation of ties in ranked observations | 636 |

Random Number Generators

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| RANDU | uniform random numbers | 260 |
| GAUSS | normal random numbers (GAUSS calls RANDU) | 426 |

MATRIX MANIPULATIONS

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| MINV | matrix inversion | 1858 |
| EIGEN | eigenvalues and eigenvectors of a real, symmetric matrix | 2540 |
| SIMQ | solution of simultaneous linear, algebraic equations | 1286 |
| GMADD | add two general matrices | 284 |
| GMSUB | subtract two general matrices | 284 |
| GMPRD | product of two general matrices | 570 |
| GMTRA | transpose of a general matrix | 354 |
| GTPRD | transpose product of two general matrices | 566 |
| MADD | add two matrices (calls LOC) | 938 |
| MSUB | subtract two matrices (calls LOC) | 938 |
| MPRD | matrix product (row into column) (calls LOC) | 944 |
| MTRA | transpose a matrix (calls MCPY) | 548 |
| TPRD | transpose product (calls LOC) | 900 |
| MATA | transpose product of matrix by itself (calls LOC) | 790 |
| SADD | add scalar to matrix (calls LOC) | 430 |
| SSUB | subtract scalar from a matrix (calls LOC) | 430 |
| SMPY | matrix multiplied by a scalar (calls LOC) | 430 |

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| SDIV | matrix divided by a scalar (calls LOC) | 466 |
| RADD | add row of one matrix to row of another matrix (calls LOC) | 534 |
| CADD | add column of one matrix to column of another matrix (calls LOC) | 546 |
| SRMA | scalar multiply row and add to another row | 438 |
| SCMA | scalar multiply column and add to another column | 426 |
| RINT | interchange two rows | 372 |
| CINT | interchange two columns | 360 |
| RSUM | sum the rows of a matrix (calls LOC) | 498 |
| CSUM | sum the columns of a matrix (calls LOC) | 498 |
| RTAB | tabulate the rows of a matrix (calls LOC, RADD) | 882 |
| CTAB | tabulate the columns of a matrix (calls LOC, CADD) | 882 |
| RSRT | sort matrix rows (calls LOC) | 1006 |
| CSRT | sort matrix columns (calls LOC and CCPY) | 950 |
| RCUT | partition row-wise (calls LOC) | 712 |
| CCUT | partition column-wise (calls LOC) | 712 |
| RTIE | adjoin two matrices row-wise (calls LOC) | 774 |
| CTIE | adjoin two matrices column-wise (calls LOC) | 742 |
| MCPY | matrix copy (calls LOC) | 406 |
| XCPY | copy submatrix from given matrix (calls LOC) | 666 |
| RCPY | copy row of matrix into vector (calls LOC) | 496 |
| CCPY | copy column of matrix into vector (calls LOC) | 496 |
| DCPY | copy diagonal of matrix into vector (calls LOC) | 390 |
| SCLA | matrix clear and add scalar (calls LOC) | 416 |
| DCLA | replace diagonal with scalar (calls LOC) | 388 |
| MSTR | storage conversion (calls LOC) | 586 |
| MFUN | matrix transformation by a function | 442 |
| RECP | reciprocal function for MFUN | 220 |
| LOC | location in compressed-stored matrix | 488 |
| CONVT | single precision — double precision conversion | 534 |
| ARRAY | vector storage — double dimensioned conversion | 666 |

OTHER MATHEMATICS

Integration

| | | |
|---|---|---|
| QUADR | integral of tabulated function | 850 |
| SMPSN | integral of given function by Simpson's rule (needs user function subprogram) | 1126 |
| RK1 | integral of first-order differential equation by Runge-Kutta method (needs user function subprogram) | 1400 |

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| RK2 | tabulated integral of first-order differential equation by Runge-Kutta method (needs user function subprogram) | 766 |
| RK3 | tabulated integral of a system of six first-order differential equations by Runge-Kutta method (needs six user function subprograms) | 2874 |

Fourier Analysis

| | | |
|---|---|---|
| FORIF | Fourier analysis of a given function (needs user function subprogram) | 982 |
| FORIT | Fourier analysis of a tabulated function | 938 |

Special Operations and Mathematical Functions

| | | |
|---|---|---|
| GAMMA | gamma function | 774 |
| LEGEN | Legendre polynomial | 588 |
| BESJ | J Bessel function | 1458 |
| BESY | Y Bessel function | 2278 |
| BESI | I Bessel function | 1286 |
| BESK | K Bessel function | 1980 |
| CELI1 | elliptic integral of the first kind | 586 |
| CELI2 | elliptic integral of the second kind | 822 |
| EXPI | exponential integral | 904 |
| SICI | Sine cosine integral | 1188 |
| CS | Fresnel integrals | 942 |

Roots of Nonlinear Equations

| | | |
|---|---|---|
| RTWIT | refine estimate of root of Wegstein's iteration (needs user function subprogram) | 850 |
| RTMIT | determine root within a range by Mueller's iteration (needs user function subprogram) | 1274 |

| Name | Function | Storage Required (Bytes) |
|---|---|---|
| RTNIT | refine estimate of root by Newton's iteration (needs user function subprogram) | 814 |

Roots of Polynomial

| | | |
|---|---|---|
| POLRT | real and complex roots of polynomial | 1916 |

Polynomial Operations

| | | |
|---|---|---|
| PADD | add two polynomials | 470 |
| PADDM | multiply polynomial by constant and add to another polynomial | 506 |
| PCLA | replace one polynomial by another | 278 |
| PSUB | subtract one polynomial from another | 472 |
| PMPY | multiply two polynomials | 502 |
| PDIV | divide one polynomial by another (calls PNORM) | 790 |
| PQSD | quadratic synthetic division of a polynomial | 378 |
| PVAL | value of a polynomial | 298 |
| PVSUB | substitue variable of polynomial by another polynomial (calls PMPY, PADDM, PCLA) | 702 |
| PCLD | complete linear division | 310 |
| PILD | evaluate polynomial and its first derivative (calls PQSD) | 398 |
| PDER | derivative of a polynomial | 346 |
| PINT | integral of a polynomial | 340 |
| PGCD | greatest common divisor of two polynomials (calls PDIV and PNORM) | 562 |
| PNORM | normalize coefficient vector of polynomial | 254 |

## APPENDIX B: ACCURACY OF SUBROUTINES

The subroutines in SSP can be broken down into three major categories from the standpoint of accuracy. They are: subroutines having little or no effect on accuracy; subroutines whose accuracy is dependent on the characteristics of the input data; and subroutines in which definite statements on accuracy can be made.

### SUBROUTINES HAVING LITTLE OR NO EFFECT ON ACCURACY

The following subroutines do not materially affect the accuracy of the results, either because of the simple nature of the computation or because they do not modify the data:

| | |
|---|---|
| TALLY | totals, means, standard deviations, minimums, and maximums |
| BOUND | selection of observations within bounds |
| SUBST | subset selection from observation matrix |
| ABSNT | detection of missing data |
| TAB1 | tabulation of data (1 variable) |
| TAB2 | tabulation of data (2 variables) |
| SUBMX | build subset matrix |
| MOMEN | first four moments |
| TTEST | tests on population means |
| ORDER | rearrangement of intercorrelations |
| AVDAT | data storage allocation |
| TRACE | cumulative percentage of eigenvalues |
| CHISQ | $\chi^2$ test for a contingency table |
| UTEST | Mann-Whitney U-test |
| TWOAV | Friedman two-way analysis of variance |
| QTEST | Cochran Q-test |
| SRANK | Spearman rank correlation |
| KRANK | Kendall rank correlation |
| WTEST | Kendall coefficient of concordance |
| RANK | rank observations |
| TIE | calculation of ties in ranked observations |
| RANDU | uniform random numbers |
| GAUSS | normal random numbers |
| GMADD | add two general matrices |
| GMSUB | subtract two general matrices |
| GMPRD | product of two general matrices |
| GMTRA | transpose of a general matrix |
| GTPRID | transpose product of two general matrices |
| MADD | add two matrices |
| MSUB | subtract two matrices |
| MPRD | matrix product (row into column) |
| MTRA | transpose a matrix |
| TPRD | transpose product |
| MATA | transpose product of matrix |
| SADD | add scalar to matrix |
| SSUB | subtract scalar from a matrix |
| SMPY | matrix multiplied by a scalar |
| SDIV | matrix divided by a scalar |
| RADD | add row of one matrix to row of another matrix |
| CADD | add column of one matrix to column of another matrix |
| SRMA | scalar multiply row and add to another row |
| SCMA | scalar multiply column and add to another column |
| RINT | interchange two rows |
| CINT | interchange two columns |
| RSUM | sum the rows of a matrix |

| | |
|---|---|
| CSUM | sum the columns of a matrix |
| RTAB | tabulate the rows of a matrix |
| CTAB | tabulate the columns of a matrix |
| RSRT | sort matrix rows |
| CSRT | sort matrix columns |
| RCUT | partition row-wise |
| CCUT | partition column-wise |
| RTIE | adjoin two matrices row-wise |
| CTIE | adjoin two matrices column-wise |
| MCPY | matrix copy |
| XCPY | copy submatrix from given matrix |
| RCPY | copy row of matrix into vector |
| CCPY | copy column of matrix into vector |
| DCPY | copy diagonal of matrix into vector |
| SCLA | matrix clear and add scalar |
| DCLA | replace diagonal with scalar |
| MSTR | storage conversion |
| MFUN | matrix transformation by a function |
| RECP | reciprocal function for MFUN |
| LOC | location in compressed-stored matrix |
| CONVT | single precision/double precision conversion |
| ARRAY | vector storage/double dimensioned conversion |
| PADD | add two polynomials |
| PADDM | multiply polynomial by constant and add to another polynomial |
| PCLA | replace one polynomial by another |
| PSUB | subtract one polynomial from another |
| PMPY | multiply two polynomials |
| PDIV | divide one polynomial by another |
| PQSD | quadratic synthetic division of a polynomial |
| PVAL | value of a polynomial |
| PVSUB | substitute variable of polynomial by another polynomial |
| PCLD | complete linear division |
| PILD | evaluate polynomial and its first derivative |
| PDER | derivative of a polynomial |
| PINT | integral of a polynomial |
| PGCD | greatest common divisor of two polynomials |
| PNORM | normalize coefficient vector of polynomial |

## SUBROUTINES WHOSE ACCURACY IS DATA DEPENDENT

The accuracy of the following subroutines cannot be predicted because it is dependent on the characteristics of the input data and on the size of the problem. The programmer using these subroutines must be aware of the limitations dictated by numerical analyses considerations. It cannot be assumed that the results are accurate simply because subroutine execution is completed. Subroutines in this category are:

| | |
|---|---|
| CORRE | means, standard deviations, and correlations |
| MULTR | multiple regression and correlation |
| GDATA | data generation |
| CANOR | canonical correlation |
| NROOT | eigenvalues and eigenvectors of a special nonsymmetric matrix |
| AVCAL | $\Sigma$ and $\Delta$ operation |
| MEANQ | mean square operation |
| DMATX | means and dispersion matrix |
| DISCR | discriminant functions |

| | |
|---|---|
| LOAD | factor loading |
| VARMX | varimax rotation |
| AUTO | autocovariances |
| CROSS | crosscovariances |
| SMO | application of filter coefficients (weights) |
| EXSMO | triple exponential smoothing |
| MINV | matrix inversion |
| EIGEN | eigenvalues and eigenvectors of a real, symmetric matrix |
| SIMQ | solution of simultaneous linear, algebraic equations |
| QUADR | integral of tabulated function |
| SMPSN | integral of given function by Simpson's rule |
| RK1 | integral of first-order differential equation by Runge-Kutta method |
| RK2 | tabulated integral of first-order differential equation by Runge-Kutta method |
| RK3 | tabulated integral of a system of six first-order differential equations by Runge-Kutta method |
| FORIF | Fourier analysis of a given function |
| FORIT | Fourier analysis of a tabulated function |
| RTWIT | refine estimate of root by Wegstein's iteration |
| RTMIT | determine root within a range by Mueller's iteration |
| RTNIT | refine estimate of root by Newton's iteration |
| POLRT | real and complex roots of polynomial |

## SUBROUTINES WITH DEFINITE ACCURACY CHARACTERISTICS

The subroutines in this section have accuracy characteristics that can be specified on an individual basis. The mathematical descriptions for many of these subroutines contain information on truncation error of a strictly theoretical nature. The actual implementation of these subroutines on System/360 results in the accuracy noted in Table 1. The standard reference for comparing the accuracy of these subroutines is M. Abramowitz, I. A. Stegun, "Handbook of Mathematical Functions", National Bureau of Standards, Washington, D. C., March 1965. It should be remembered that in System/360 single-precision floating point, there are just over six significant figures.

Table 1. Subroutines with Definite Accuracy Characteristics

| Name | Function | Allowable Parameter Range | Range Checked with Standard Reference | Maximum Error Compared to Standard Reference | Expected Maximum Error Beyond Standard Reference Range |
|---|---|---|---|---|---|
| GAMMA | gamma function | X not within $10^{-6}$ of zero or negative integer | $1.0 \leq X \leq 57$ Values beyond 57 exceed the range of the machine | one in fifth significant figure | same (see note) |

Note: For arguments near zero or a negative integer, the maximum error will be equal to the number of zeros to the right of the decimal point when X is subtracted from its nearest integer value.

| Name | Function | Allowable Parameter Range | Range Checked with Standard Reference | Maximum Error Compared to Standard Reference | Expected Maximum Error Beyond Standard Reference Range |
|---|---|---|---|---|---|
| LEGEN | Legendre polynomial | $-1.0 \leq X \leq 1.0$<br>$N \geq 0$ | $0.0 \leq X \leq 1.0$<br>$0 \leq N \leq 10$ | one in fifth significant figure | (See mathematical description page for LEGEN) |
| BESJ | J Bessel function | $X > 0.0$<br>$N \geq 0$<br>$N < 20+10x-\frac{x^2}{3}$ for $X \leq 15$<br>$N < 90 + \frac{x}{2}$ for $X > 15$ | $0.0 < X \leq 100.0$<br>$0 \leq N \leq 100$ | same as specified error up to hardware significance ($10^{-6}$) | same within range of hardware ($10^{-75}$ - $10^{+75}$) |
| BESY | Y Bessel function | $X > 0.0$<br>$N \geq 0$ | $0.0 < X \leq 100.0$<br>$0 \leq N \leq 100$ | one in fifth significant figure | same within range of hardware ($10^{-75}$ - $10^{+75}$) |
| BESI | I Bessel function | $X \geq 0$<br>$N \geq 0$ | $0.0 \leq X \leq 100.0$<br>$0 \leq N \leq 100$ | five in the fifth significant figure for values within range of hardware ($10^{-75}$ -$10^{+75}$) | same |
| BESK | K Bessel function | $X > 0$<br>$N \geq 0$ | $0.0 < X \leq 100.0$<br>$0 \leq N \leq 100$ | one in the fifth significant figure for values within range of hardware ($10^{-75}$ -$10^{+75}$) | same |
| CELI1 | complete elliptic integral of first kind | $-1.0 \leq AK \leq 1.0$ | $-1.0 \leq AK \leq 1.0$ | one in the fifth significant figure | Not applicable |
| CELI2 | complete elliptic integral of second kind | $-1.0 \leq AK \leq 1.0$ | $-1.0 \leq AK \leq 1.0$ | one in the fifth significant figure | Not applicable |

Table 1. Subroutines with Definite Accuracy Characteristics (continued)

| Name | Function | Allowable Parameter Range | Range Checked with Standard Reference | Maximum Error Compared to Standard Reference | Expected Maximum Error Beyond Standard Reference Range |
|---|---|---|---|---|---|
| EXPI | exponential integral | $X \geq -4.0$ | $-4.0 \leq X \leq 10.0$ | one in the fifth significant figure | same |
| | Note: For $X > 0$, integral is defined as $E_1$ (X) in standard reference<br>For $X < 0$, integral is defined as $-E_i$ (-X) in standard reference | | | | |
| SICI | Sine and cosine integrals | No restriction | $0 \leq X \leq 10.0$ | one in fifth significant figure | same |
| | Note: These two integrals are defined as $s_i$ and $C_i$ in the standard reference | | | | |
| CS | Fresnel integrals | No restriction (the absolute value of the argument is used) | $0 \leq X < 40.0$ | one in fifth significant figure | same |
| | Note: These two integrals are defined as $C_2$ and $S_2$ in the standard reference | | | | |

## APPENDIX C: LIST OF DOUBLE-PRECISION SUBROUTINES AND SAMPLE PROGRAMS

The following SSP subroutines and sample programs include information on modification for double precision as part of the subroutine description:

SUBROUTINES

CORRE

ORDER

MULTR

GDATA

CANOR

NROOT

ABDAT

AVCAL

MEANQ

DMATX

DISCR

TRACE

LOAD

VARMX

MINV

EIGEN

QUADR

SMPSN

RK1

RK2

RK3

POLRT

SAMPLE MAIN PROGRAMS

Multiple Regression

REGRE

Polynomial Regression

POLRG

Canonical Correlation

MCANO

Analysis of Variance

ANOVA

Discriminant Analysis

MDISC

Factor Analysis

FACTO

Numerical Quadrature

QDINT

Runge-Kutta Integration

RKINT

Real and Complex Roots of Real Polynomial

SMPRT

## PART II

## INTRODUCTION

### OVERVIEW

This portion of the manual describes the additional capabilities to be provided in the System/360 Scientific Subroutine Package Version 2 beyond that contained in the currently available Scientific Subroutine Package (360A-CM-03X). Version 2, which is not yet available but will be available in the near future, will consist of all the subroutines in SSP/360 plus the additional subroutines described below. Descriptions of the current SSP/360 are found in Part I of this manual and in the Programmer's Manual (H20-0205). This new set of subroutines, like its predecessors, is a collection of input/output-free computational building blocks that can be combined with a user's input/output or computational routines to meet his needs. The package can be applied to the solution of many problems in industry, science, and engineering.

### CONTENT

SSP Version 2 is a major extension of SSP/360 in the area of mathematics. Individual subroutines can be used to carry out the listed functions in the following areas:

- Generalized elliptic integrals
- Chebyshev polynomial operations
- Hermite polynomial operations
- Laguerre polynomial operations
- Legendre polynomial operations
- Polynomial economization
- Solution of linear least squares
- Roots of a polynomial by quotient-difference algorithm
- Unconstrained minimum of a function
- Interpolation
- Integration by the following methods:
  Trapezoid, Simpson's Hermitian, Gaussian, Gaussian Laguerre, Gaussian Hermite, Generalized Gaussian Laguerre, and Trapezoid with Romberg's extrapolation.
- Solution of N first-order ordinary differential equations by the following methods:
  Runge Kutta with given initial values.
  Hamming Modified Predictor-Corrector with given initial values.
  Adjoint Equations for linear system with linear boundary conditions.
- Fourier Analysis by Cooley-Tukey Algorithm

### CHARACTERISTICS

Some of the characteristics of this extension to SSP/360 are:

- All subroutines are free of input/output statements.
- Subroutines do not contain fixed maximum dimensions for the data arrays named in their calling sequence.
- All subroutines are written in FORTRAN.
- All subroutines are uniformly documented.

### DESIGN PHILOSOPHY

The algorithms in this extension have been chosen in order to provide new capabilities or more advanced versions of capabilities currently available in SSP/360. For example, the currently available SSP/360 contains a subroutine for the Fourier analysis of real tabulated data (FORIT). Version 2 includes a new Fourier subroutine which is much faster. In addition, another subroutine provided in SSP II will do a Fourier analysis on real complex three-dimensional data. Both of the new subroutines use the Cooley-Tukey algorithm. These subroutines are of course much larger than FORIT but will still fit easily within a 32K System/360.

## SUBROUTINES

### GENERAL REMARKS

Below are listed the subroutines to be added to SSP/360 in order to form Version 2:

### SUBROUTINE LIST

Special Operations and Mathematical Functions

ELI1 — generalized elliptic integral of the first kind
ELI2 — generalized elliptic integral of the second kind
JELF — Jacobian elliptic functions
CNP — value of N-th Chebyshev polynomial
CNPS — value of series expansion in Chebyshev polynomials
TCNP — transform series expansion in Chebyshev polynomials to a polynomial
CSP — value of N-th shifted Chebyshev polynomial
CSPS — value of series expansion in shifted Chebyshev polynomials
TCSP — transform series expansion in shifted Chebyshev polynomials to a polynomial

HEP — value of Hermite polynomial

HEPS — value of series expansion in Hermite polynomials

THEP — transform series expansion in Hermite polynomials to a polynomial

LAP — value of Laguerre polynomial

LAPS — value of series expansion in Laguerre polynomials

TLAP — transform series expansion in Laguerre polynomials to a polynomial

LEP — value of Legendre polynomial

LEPS — value of series expansion in Legendre polynomials

TLEP — transform a series expansion in Legendre polynomials to a polynomial

Polynomial Operations

PECN — economization of a polynomial for symmetric range

PECS — economization of a polynomial for unsymmetric range

Solution of Linear Equations

GELG — system of general simultaneous linear equations by Gauss elimination

LLSQ — solution of linear least squares problems

Solution of Nonlinear Equations

PRQD — roots of a real polynomial by quotient-difference algorithm with displacement

FMFP — unconstrained minimum of a function of several variables — Davidson method

FMCG — unconstrained minimum of a function of several variables — conjugate gradient method

Approximation, Interpolation and Table Construction

ALI — Aitken-Lagrange interpolation

AHI — Aitken-Hermite interpolation

ACFI — continued fraction interpolation

ATSG — table selection out of a general table

ATSM — table selection out of a monotonic table

ATSE — table selection out of an equidistant table

Numerical Integration and Differentiation

QTFG — integration of monotonically tabulated function by trapezoidal rule

QTFE — integration of equidistantly tabulated function by trapezoidal rule

QSF — integration of equidistantly tabulated function by Simpson's rule

QHFG — integration of monotonically tabulated function with first derivative by Hermitian formula of first order

QHFE — integration of equidistantly tabulated function with first derivative by Hermitian formula of first order

QHSG — integration of monotonically tabulated function and second derivatives by Hermitian formula of first order

QHSE — integration of equidistantly tabulated function with first and second derivatives by Hermitian formula of second order

QG2- QG10 — integration of a given function by Gaussian quadrature formulae

QL2- QL10 — integration of a given function by Gaussian-Laguerre quadrature formulae

QH2- QH10 — integration of a given function by Gaussian-Hermite quadrature formulae

QLG2-- QLG10 — integration of a given function by generalized Gaussian-Laguerre quadrature formulae

QATR — integration of a given function by trapezoidal rule together with Romberg's extrapolation method

Solution of Ordinary Differential Equations

RKGS — solution of system of first-order ordinary differential equations with given initial values by the Runge-Kutta method

HPCG — solution of general system of first-order ordinary differential equations with given initial values by Hamming's modified predictor-corrector method

HPCL — solution of linear system of first-order ordinary differential equations with given initial values by Hamming's modified predictor-corrector method

LBVP — solution of system of linear first-order ordinary differential equations with linear boundary conditions by method of adjoint equations

Fourier Analysis

RHARM — finds the Fourier coefficients of one-dimensional real data using the Cooley-Tukey algorithm.

HARM — performs discrete complex Fourier transforms on a complex three dimensional array using the Cooley-Tukey algorithm.

## OVERALL RULES OF USAGE

### SUBROUTINE USAGE

All subroutines in Version 2 are entered by means of the standard FORTRAN CALL statement. These subroutines are purely computational in nature and do not contain any references to I/O devices. The user must therefore furnish, as part of his program, whatever I/O and other operations are necessary for the total solution of his problem. In addition, the user must define by DIMENSION statements all matrices to be operated on by SSP subroutines as well as those matrices utilized in his program. The subroutines contained in SSP are no different from any user-supplied subroutine. All of the normal rules of FORTRAN concerning subroutines must, therefore, be adhered to, except that the dimensioned areas in the SSP subroutine are not required to be the same as those in the calling program.

### MATRIX OPERATION

#### Variable Dimensioning

Those subroutines that deal with matrices can operate on any size array, limited, in most cases, only by the available core storage and numerical analysis considerations. The subroutines do not contain fixed maximum dimensions for data arrays named in their calling sequence. The variable dimension capability has been implemented in SSP by using a vector storage approach. Under this approach, each column of a matrix is immediately followed in storage by the next column. Vector storage and two-dimensional storage result in the same layout of data in core, so long as the number of rows and columns in the matrix is the same as in the user's dimension statement. If, however, the matrix is smaller than the dimensioned area, the two forms of storage are not compatible. A subroutine called ARRAY is available in SSP to change from one form of storage to the other. In addition, a subroutine called LOC is available to assist in referencing elements in an array stored in the vector fashion.

## REQUIRED SYSTEM

### PROGRAMMING SYSTEM

System/360 Basic Programming Support FORTRAN Compiler (tape). However, the subroutines of SSP/360 Version 2 should compile and execute with any System/360 FORTRAN compiler which is capable of accepting Basic Programming Support FORTRAN statements.

### MACHINE CONFIGURATION

A basic requirement is a System/360 configuration suitable for the FORTRAN compiler chosen. The machine configuration required for any given problem depends on the number of subroutines used, the size of the compiled subroutine, the size of the compiled main program, the size of the control program, and the data storage requirements.

## REFERENCE MATERIAL

System/360 Scientific Subroutine Package (360A-CM-03X) Programmer's Manual (H20-0205). This manual contains sufficient information to permit the reader to understand and use all of the subroutines in the currently available Scientific Subroutine Package (SSP/360).

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IBM

International Business Machines Corporation
Data Processing Division
112 East Post Road, White Plains, N.Y. 10601
(USA Only)

IBM World Trade Corporation
821 United Nations Plaza, New York, New York 10017
(International)